Name: Mahyar Pirayesh ____________ Date: ________________________

HW Math 12 Honors: Section 6.4 Polynomials with Complex Roots:

1. When given a polynomial in the form $y = x^4 + 3x^3 + 5x^2 + 12x + 4$, how many roots are there? If some of the roots are complex, what do you know about these complex roots?

4 roots. Complex roots must appear in conjugate pairs if all coefficients are real numbers.

2. Suppose you are given a polynomial $y = x^4 + Ax^2 + Bx + C$, where all the coefficients are real numbers. If one of roots is $z = 2+3i$ then find another root.

$z_2 = 2-3i$ //

3. Given the polynomial $y = x^3 + x^2 + Bx + C$, where one root is $z = 1+\sqrt{2}i$, what are the other two roots and what is the value of "C"?

$z_2 = 1-\sqrt{2}i$ By way of vieta: $-1 = 1 - \cancel{\sqrt{2}i} + 1 + \cancel{\sqrt{2}i} + r \Rightarrow r = -3$

$B = (1-\sqrt{2}i)(1+\sqrt{2}i) + (1-\sqrt{2}i)(-3) + (1+\sqrt{2}i)(-3) = \cancel{3} - \cancel{3} + \cancel{3\sqrt{2}i} - 3 - \cancel{3\sqrt{2}i} = -3$ //

$C = -(-3)(1-\sqrt{2}i)(1+\sqrt{2}i) = \boxed{9}$ //

4. Use synthetic division or long division to find the quotient and remainder.
$\left(x^4 + 16x^3 + 67x^2 + 63x - 70\right) \div (x+10)$

$x^4 + 16x^3 + 67x^2 + 63x - 70 = (x+10)(\underbrace{x^3 + 6x^2 + 7x - 7}_{\text{quotient}})$ Rem $= 0$ //

5. How can you tell if a polynomial function $P(x) = ax^3 + bx^2 + cx + d$ is divisible by $(x-e)$? What does it mean if the polynomial is divisible by the binomial factor ? Explain:

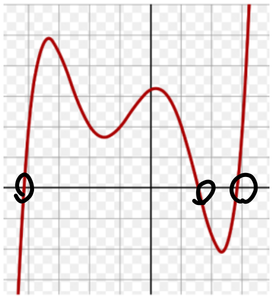
You can tell by long division (divisible if remainder is 0)

If divisible, it means that "e" is a root of the polynomial.

6. Given the polynomial function, which of the following will give you the sum of all the coefficients?
$P(x) = x^n + Bx^{n-1} + Cx^{n-2} + Dx^{n-3} + \ldots + E$:

[✓ i) $P(1)$] ii) $P(2)$ iii) $P(1)+1$ iv) $P(0)+1$

7. The polynomial function below has a degree of 7 and has it's roots shown. Suppose all roots are single roots, how many complex roots does it have? Explain and justify your answer:



3 real roots

$7 - 3 = 4$ complex roots //

8. Factor each of the polynomials and then solve for all roots:

a) $\left(x^2+1\right)\left(x^2+9\right)=0$ $x^2=-1$ $\quad x^2=-9$ $\boxed{x=\pm i}$ $\quad \boxed{x=\pm 3i}$	b) $\left(x^2+10\right)\left(x^2+2x+4\right)=0$ $x^2=-10$ $\quad x=\frac{-2\pm\sqrt{4-16}}{2}$ $\boxed{x=\pm\sqrt{10}\,i}$ $\quad \boxed{x=-1\pm\frac{\sqrt{10}}{2}i}$	c) $x^4+12x^2+35=0$ $A^2+12A+35=0$ $(A+7)(A+5)=0$ $x^2=-7$ $\quad x^2=-5$ $\boxed{x=\pm\sqrt{7}\,i \quad x=\pm\sqrt{5}\,i}$
d) $\left(x-1+3i\right)\left(x+1+3i\right)=0$ $\boxed{x_1=1-3i \quad x_2=-1-3i}$	e) $x^3+3x^2+x-5=0$ $x_1=1$ $1+3+1-5=0$ x^2+4x+5 $x-1\,\overline{)\,x^3+3x^2+x-5}$ $-(x^3-x^2)$ $4x^2+x-5$ $-(4x^2-4x)$ $5x-5$ $5x-5$ 0 $=(x-1)(x^2+4x+5)$ $\boxed{x_1=1}$ $\quad x_2=\frac{-4\pm\sqrt{16-20}}{2}=\boxed{-2\pm 2i}$	f) $0=x^3+3x^2+4x+12$ $x^2(x+3)+4(x+3)=0$ $(x+3)(x^2+4)=0$ $\boxed{x_1=-3 \quad x_2=\pm 2i}$
g) $9x^3+2x+1=0$ $\boxed{r_1=-\frac{1}{3}}$ $9x^2-3x+3$ $x+\frac{1}{3}\,\overline{)\,9x^3+2x+1}$ $-(9x^3+3x^2)$ $-3x^2+2x+1$ $-(-3x^2-x)$ $3x+1$ $3x+1$ 0 $9x^3+2x+1=(x+\frac{1}{3})(9x^2-3x+3)$ $=3(x+\frac{1}{3})(3x^2-x+1)$ $x=\frac{1\pm\sqrt{1-12}}{6}$ $r_{2,3}$: $\boxed{x=\frac{1\pm 11i}{6}}$	h) $x^4+9x^2+20=0$ $A^2+9A+20=0$ 4 5 $(A+4)(A+5)=0$ $(x^2+4)(x^2+5)=0$ $\boxed{x=\pm 2i \quad x=\pm\sqrt{5}\,i}$	i) $x^4+5x^2-24=0$ $x^2=A$ $A^2+5A-24=0$ $(A-3)(A+8)=0$ $A=3$ $\quad A=-8$ $x^2=3$ $\quad x^2=-8$ $\boxed{x_1=\sqrt{3} \quad x_2=-\sqrt{3}}$ $\quad \boxed{x_3=2\sqrt{2}\,i \quad x_4=-2\sqrt{2}\,i}$

j) $x^4+x^3+7x^2+9x-18=0$	k) $8x^4+50x^3+43x^2+2x-4=0$	L) $4x^4-4x^3+13x^2-12x+3=0$
$r_1=1$ $x^3+2x^2+9x+18$ $x-1\overline{)x^4+x^3+7x^2+9x-18}$ $-(x^4-x^3)$ $2x^3+7x^2+9x-18$ $-(2x^3-2x^2)$ $9x^2+9x-18$ $-(9x^2-9x)$ $18x-18$ $r_2=-2$ x^2+9 $x+2\overline{)x^3+2x^2+9x+18}$ $-(x^3+2x^2)$ $9x+18$ $9x+18$ $x^4+x^3+7x^2+9x-18=(x-1)(x+2)(x^2+9)$	$r_1=-\frac{1}{2}$ $r_2=\frac{1}{4}$ $(2x+1)(4x-1)(x^2+6x+4)=0$ $\boxed{x=-\frac{1}{2},\frac{1}{4},-3\pm\sqrt{5}}$	$r_1=\frac{1}{2}$ $r_2=\frac{1}{2}$ $(2x-1)(2x-1)(x^2+3)=0$ $\boxed{r=\frac{1}{2},\pm\sqrt{3}i}$

$\boxed{r=1,-2,3i,-3i}$

9. Given that the polynomial $f(x)=12x^5-20x^4+19x^3-6x^2-2x+1$ has roots at $x=\frac{1}{2}$ and $x=\frac{1}{3}$, find the other complex roots.

$f(x)=(2x-1)^2(3x+1)(\underline{x^2-x+1})$ by long division

$$x=\frac{1\pm\sqrt{1-4}}{2}=\boxed{\frac{1\pm\sqrt{3}i}{2}}$$

10. Given the function, how many roots are there? Find all the solutions and then indicate all NPV's if there are any.

$$\frac{(z^5+z^4+z^3+z^2+z^1+1)(z^7+1)}{z^5+1}=0 \Rightarrow \frac{\left(\frac{z^6-1}{z-1}\right)(z^7+1)}{z^5+1}=0 \Rightarrow$$

$z^6=1$

$z\neq1$

$z^7=-1$

$z^5\neq-1$

$z=e^{60^\circ ki}$; $k\in\{1,2,4,5\}$

$z=e^{\frac{360}{7}^\circ mi}$; $m\in\{1,2,3,4,5,6\}$

NPV: $z\neq1$, $z\neq e^{72^\circ ni}$; $n\in\{0,1,2,3,4\}$

11. Given the polynomial $P(x)=x^3+3x^2+Bx+C$ with real coefficients and a complex root at $z=1-4i$, find the coefficients "B" and "C".

$z_1=1-4i$

$z_2=\overline{z_1}=1+4i$

$z_1+z_2+r_3=-3$ by vieta sums

$2+r_3=-3\Rightarrow r_3=-5$

$$\boxed{\begin{array}{l}B=z_1z_2+z_1r_3+z_2r_3=7\\ C=z_1z_2r_3=-85\end{array}}$$

12. Given the polynomial $P(x)=x^3+Ax^2+Bx+24$ with real coefficients and a complex root at $z=3+2i$, find the coefficients "A" and "B".

$r_2=3-2i$

$24=r_1r_2r_3=(3+2i)(3-2i)r_3$

$24=13r_3$

$r_3=\frac{24}{13}$

$-A=r_1+r_2+r_3$

$A=6+\frac{24}{13}=\boxed{\frac{102}{13}}$

$B=r_1r_2+r_1r_3+r_2r_3=13+\frac{144}{13}=\boxed{\frac{303}{13}}$

13. There are nonzero integers “a”, “b”, “r”, and “s” such that the complex number $r+si$ is a zero of the polynomial $P(x)=x^3-ax^2+bx-65$. For each possible combination of “a” and “b”, let $p_{a,b}$ be the sum of the zeroes of $P(x)$. Find the sum of the $p_{a,b}'s$ for all possible combinations of “a’ and ‘b” [2013 AIME]

14. Let “S” be the set of all polynomials of the form z^3+az^2+bz+c, where “a”, “b”, and “c” are integers. Find the number of polynomials in “S” such that each of its roots “z” satisfies either $|z|=20$ or $|z|=13$ [2013 AIME II]

15. Let $P(x)$ be a polynomial with integer coefficients that satisfies $P(17)=0$ and $P(24)=17$. Given that $P(n)=n+3$ has two distinct integer solutions n_1 and n_2, find the product $n_1 \times n_2$ [AIME 2005]

16. For certain real values “a”, “b”, “c” and “d’, the equation $x^4+ax^3+bx^2+cx+d=0$ has four non-real roots. The product of two of the non real roots is $13+i$ and the sum of the other two non real roots is $3+4i$. Find the value of “b”. [Hint: Since all the coefficients are real, the roots must come in conjugate pairs] Aime 1995

17. The polynomial $P(x)=(1+x+x^2+....x^{17})^2-x^{17}$ has 34 complex roots of the form $z_k=r_k\left[\cos(2\pi a_k)+i\sin(2\pi a_k)\right],\ k=1,2,3,....34$ with $0<a_1\le a_2\le....\le a_{34}\le 1$ and $r_k>0$. Given that $a_1+a_2+a_3+a_4+a_5=\dfrac{m}{n}$, where "m' and "n" are relatively prime positive integers, find "m" and "n" [2004 AIME 1]

18. The polynomial $f(z)=az^{2018}+bz^{2017}+cz^{2016}$ has real coefficients not exceeding 2019, and $f\left(\dfrac{1+\sqrt{3}i}{2}\right)=2015+2019\sqrt{3}i$. Find the value of $f(1)$ [AIME 2019]

SOL at 6.2 #16

19. Let "P" be the product of the roots of $z^6+z^4+z^3+z^2+1=0$ that have a positive imaginary part, and suppose that $P=r(\cos\theta°+i\sin\theta°)$ where $r>0$ and $0°<\theta<360°$. Find θ [AIME 1996]